Introduction :

Probability theory is a branch of mathematics that deals with the properties of certain structures that model phenomena in which 'chance' intervenes.

of certain structures modelling phenomena in which "chance" is involved. As an abstract

abstract mathematical theory, it is based on axiomatics and develops autonomously from physical reality.

physical reality. Only the names of the concepts used (events, variables, etc.) refer to experience.

Probability theory enables certain random phenomena to be modelled efficiently and studied theoretically.

How does it relate to statistics, which is based more on the observation of concrete phenomena?

phenomena? Broadly speaking, there are three: firstly, the data observed are often imprecise are often imprecise and fraught with error. The probabilistic model then makes it possible to represent the deviations between 'true' values and observed values as random variables observed values.

The term probability has several meanings: historically, it comes from the Latin probabilitas, and refers to the opposite of the concept of certainty; it is also an evaluation of the probable nature of an event, meaning that a value can be used to represent its degree of certainty; recently, probability has become a mathematical science and is called probability theory or, more simply, probability; finally, a doctrine is also known as probabilism.

The probability of an event is a real number between 0 and 1. The greater the number, the greater the risk, or chance, of the event occurring. The scientific study of probability is relatively recent in the history of mathematics. The study of probability has undergone numerous developments since the eighteenth century, thanks to the study of the random and partly unpredictable nature of certain phenomena.

The probability of an event characterises the possibility of that event occurring,

Chap1 : Introduction to calculating probabilities

1.1 combinatorial analysis:

Combinatorial analysis is concerned with studying and counting the various types of

groupings that can be made from finite sets.

Combinatorial analysis comprises a set of methods that make it possible to determine the number of all possible outcomes of a particular experiment.

(to count the different arrangements that can be formed from a set of

E.g. the components of a vehicle number plate.

Example. Numbers from a lottery draw.

Exemple 1 : Consider a set with two elements fa; bg. With two draws without repetition, we can obtain $\{a, b\}$ or $\{b, a\}$; With two repeated tooks, we can obtain$\{a, a\}\{a, b\}\{b, a\}\{b, b\}$.

This corresponds to a random selection.

Exemple 2 : Let's take a set of 6-sided dice (discernible elements) numbered by $\{1,2,3,4,5,6\}$.

After 3 throws, we obtain the realisation A = (2; 5; 1); we repeat the throws and obtain B =(5; 1; 2). A and B are equivalent if we consider that the arrangements are non-ordered.

On the other hand, they are not equivalent if we are in the context of an ordered disposition.

La valeur Factorielle(n), notée n! est définie par $n! = 1 \cdot 2 \cdots n = \prod_{i=1}^{n} i$

By convention 0! = 1. We can also use a recursive definition

$$n! = n \cdot (n-1)!$$

The enumeration methods will be classified according to 3 categories: - permutations - arrangements – combinations

1.1.1. Arrangement with repetition :

Let Ω be a set with n elements. If we have to choose p elements from n in an ordered arrangement (the places are distinct) and with repetition (we can choose the same element several times), we say that we have an arrangement of p elements from n. The number of arrangements with repetition is n^p.

N.B. In this case, it is possible that p > n

Eg 1:
{1,2,3}. arrangements with repetitions of two elements of the set are :
{1,2}.{1,3}.{2,3}.{2,1}.{3,1}.{3,2}.{1,1}.{2,2}.{3,3}.
$3^2=9$
Ex 2: An urn contains 1R, 1B, 1N and 1V. 2 successive tosses are
successive draws. How many possible outcomes are there?

$$\underbrace{\begin{array}{cccc} RR & RB & RN & RV \\ BB & BR & BN & BV \\ NN & NR & NB & NV \\ VV & VR & VB & VN \end{array}}$$

4^2=16

1.1.2 Arrangement without repetition :

Let Ω be a set with card (Ω) = n . A sample of size p (p < n) is formed, the arrangement is ordered and without repetition. We say that we have a non-repeating arrangement of p elements among n. The number of p arrangements of a set with n elements is :

$$A_n^P = \frac{n!}{(n-p)!}$$

Example:

{1,2,3}. The arrangements without repetitions of two elements of the set are :$\{1,2\}.\{1,3\}.\{2,3\}.\{2,1\}.\{3,1\}.\{3,2\}.$

$$A_3^2 = \frac{3!}{(3-2)!} = 3!$$

1.1.3 Permutation without repetition :

This is a non-repeating arrangement of n elements from n

$$P_n = A_n^n = \frac{n!}{(n-n)!} = n!$$

Example1: {1,2,3}. the permutations without repetitions of two elements of the set are :

$$\{1,2,3\}.\{2,1,3\}.\{1,3,2\}.\{2,3,1\}.\{3,1,2\}.\{3,2,1\}.$$

$$P_n = A_3^3 = \frac{3!}{0!} = 3!$$

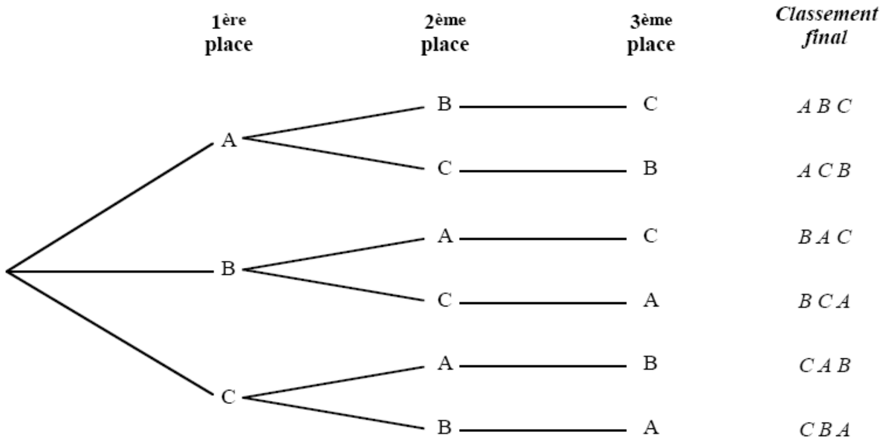
$$3! = 3 \times 2 = 6$$

A permutation of the elements of Ω is an exhaustive non-remitted choice of the elements of Ω, taking into account the order of the choice.

Example 2:

Suppose three teams are taking part in a tournament in which first, second and third place are determined. To make it easier to identify the teams, we'll refer to them as A, B and C.

Let's find out how many different ways there are to rank these 3 teams. We can illustrate this reasoning with a tree diagram.



1.1.4 Permutation with repetition :

We call permutation with repetition of p elements where n are distinct (n < p),
an ordered arrangement of the set of these p elements where the first appears p1 times, the second p2 times, etc., such that p1 + p2 + ... + pn = p. The number of permutations with repetitions is

$$= \frac{p!}{p_1!\,p_2!\,\ldots.\,p_n!}$$

Example 1: [Number of anagrams for the word MATHEMATICS]: we can see that by swapping the two letters A, the word remains identical, but by transposing the letters É and E we get a different word (M :2 ;A :2 ;T :2 ;H :1 ; ;I :1 ;Q :1 ;U :1 ;E :2)

Anagrammes=$\frac{12!}{2!\times2!\times2!\times2!\times1!\times1!\times1!\times1!\times1!} = 29937600$

1.1.5 Combination without repetition :(Set)

Consider a set Ω composed of n elements, all of which are discernible. We form a sample of size p. If the disposition is non-ordered and without repetition, we say that we have a combination without repetition of p elements among n. The number of such combinations is C_n^p

$$C_n^P = \frac{n!}{p!\,(n-p)!}$$

Example:

The combination without repetition of P=2 elements among n elements of the set {1,2,3} are

$C_3^2 = \frac{3!}{2!(3-2)!} = 3$

{1,2}, {1,3} , {2,3}

Properties :

1. $C_n^0 = C_n^n = 1$
2. $C_n^p = C_n^{n-p}$ (complementary)
3. $C_n^p = C_{n-1}^p + C_{n-1}^{p-1}$ (triangle of Pascal)
4. $C_n^p = \frac{A_n^p}{p!}$

$C_n^p = C_{n-1}^p + C_{n-1}^{p-1}$:

$$\begin{aligned} C_{n-1}^p + C_{n-1}^{p-1} &= \frac{(n-1)!}{p!(n-p-1)!} + \frac{(n-1)!}{(p-1)!(n-p)!} \\ &= \frac{(n-1)!\cdot(n-p)}{p!(n-p)!} + \frac{p\cdot(n-1)!}{p!(n-p)!} \\ &= \frac{n\cdot(n-1)!}{p!(n-p)!} = C_n^p \end{aligned}$$

Pair formula :

$$(a+b)^n = \sum_{p=0}^{n} C_n^p \cdot a^p \cdot b^{n-p}.$$

1				
1	1			
1	2	1		
1	3	3	1	
1	4	6	4	1

Triangl of Pascal

Example 15 : $(x+y)^3 = \sum_{k=0}^{3} C_3^k x^k y^{3-k} = C_3^0 y^3 + C_3^1 x^1 y^2 + C_3^2 x^2 y^1 + C_3^3 x^3 = y^3 + 3xy^2 + 3x^2 y + x^3$

1.1.6 Combination with repetition :

This is an unordered arrangement of p elements, to be chosen from n discernible elements, with repetition. The number of combinations with repetitions of n objects taken p by p is :

$$K_n^p = C_{n+p-1}^P$$

Example:

The combination with repetition of P =2 elements among n elements of the set {1,2,3} are :

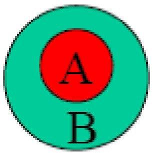
{1,2}. {1,3}. {2,3}. {1,1}. {2,2}. {3,3}.

II Notion of set theory

Subset:

Let A and B be two sets such that: $A \subset \Omega$, $B \subset \Omega$

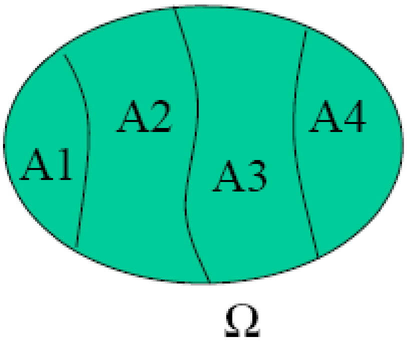
We say that A is included in B, we note $A \subset B$ if each element of $A \in B$



A and B are said to be equal, A=B if $A \subset B$ and $B \subset A$ if $A \subset B$.
if $A \subset B$ so $card(A) \leq card(B)$

Complete system of events : Let A1 , A2 , ... An n events. We say that (A1 , A2 , ... , An) constitute a Complete system of events if they form a partition of Ω :

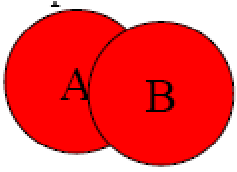
If their union is the certain event Ω



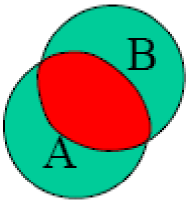
$$\boxed{\bigcup_{p=1}^{n} A_p = \Omega}$$

Union and intersection :

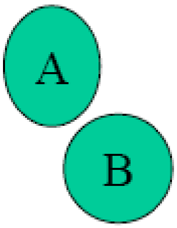
$$A \cup B = \{w \in A \text{ or } w \in B\}$$



$$A \cap B = \{w \in A \text{ and } w \in B\}$$



We interpret the event AUB as being (A or B come realised), and A⋂B as being (A and B come realised), Two events A and B are said to be disjoint if they have no elements in common, i.e. if A⋂B=∅, where ∅ is the empty set.



Ex: life and death

Notion :

$$\bigcap_{i=1}^{n} Ai = A_1 \cap A_2 \cap \ldots \ldots \cap A_n$$

$$\bigcup_{i=1}^{n} Ai = A_1 \cup A_2 \cup \ldots \ldots \cup A_n$$

Proposition :

- Card(AUB)=card(A) + card (B) – card (A∩B)
- Intersection and reunion are laws of internal composition.

  $A\cup B \in P(\Omega) \text{et } \mathrm{A} \cap \mathrm{B} \in \mathrm{P}(\Omega)\ \forall \mathrm{A}, \mathrm{B}\ \in \mathrm{P}(\Omega)$
- Commutativity $A\cup B = B\cup A\ et\ \mathrm{A} \cap \mathrm{B} = \mathrm{B} \cap \mathrm{A}$
- Associativity : $\mathrm{A} \cap (\mathrm{B} \cap \mathrm{C}) = (\mathrm{A} \cap \mathrm{B}) \cap \mathrm{C} = \mathrm{A} \cap \mathrm{B} \cap \mathrm{C}$

  $$\mathrm{A} \cup (\mathrm{B} \cup \mathrm{C}) = (\mathrm{A} \cup \mathrm{B}) \cup \mathrm{C} = \mathrm{A} \cup \mathrm{B} \cup \mathrm{C}$$
- Neutral event:

  $\mathrm{A} \cup (\emptyset) = \mathrm{A}$

  $\mathrm{A} \cap (\Omega) = \mathrm{A}$

$$\mathrm{A} \cap \left(\bigcup_{\mathrm{i}=1}^{\mathrm{n}} \mathrm{Bi}\right) = \bigcup_{\mathrm{i}=1}^{\mathrm{n}} (\mathrm{A} \cap \mathrm{Bi})$$

In particular $\mathrm{A} \cap (\mathrm{B} \cup \mathrm{C}) = (\mathrm{A} \cap \mathrm{B}) \cup (\mathrm{A} \cap \mathrm{C})$

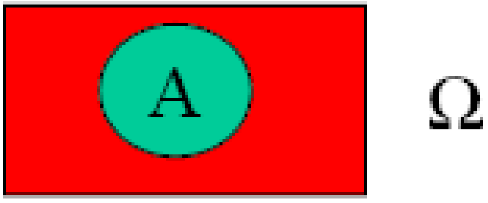
$$A \cup \left(\bigcap_{i=1}^{n} Bi \right) = \bigcap_{i=1}^{n} (A \cup Bi)$$

En particulier

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

Complementary set :

def: let Ω be a set and A a subset of Ω, we call the complementary set of A in Ω which we denote A or $C_{\Omega}A$ or A^c , the set



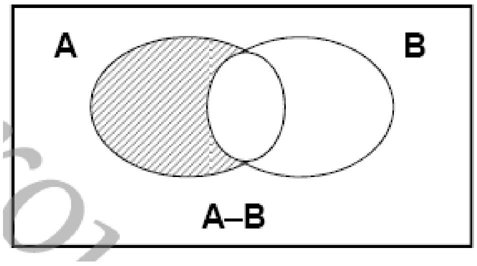
Subtraction of sets :

Definition:

Let A,B be two sets of Ω, we call the difference of sets A and B which we note A\B

We note

A\B= $\{\omega \in A \ and \ \omega \notin B\}$

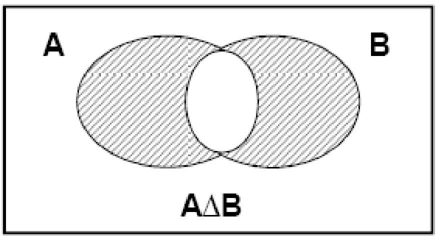


Symmetric difference:

The symmetric difference of two sets A and B is the set whose elements belong to either A or B but not a (A∩B) .

We note $A\Delta B$

$A\Delta B = \{x \in \Omega \ / \ \ x \in A \cup B and \ x \notin A \cap B\}$



Example :

Considérons deux ensemble A et B

A est l'ensemble des cadres

B est l'ensemble des titulaires d'une administration

A ∆B est l'ensemble qui réunit les cadres non titulaire et les titulaires non cadres.

Introduction to the concept of probability

2.1. Random experiment - event

Definition 01

An experiment is said to be random if the outcome cannot be completely predicted.

We denote Ω the set of all possible values that the results of this experiment can take, called the "universe".

Example:

1° / Toss a coin: $\boldsymbol{\Omega} = \{\boldsymbol{P}, \boldsymbol{F}\}$

2° / Throwing a cubic die: $\boldsymbol{\Omega} = \{\mathbf{1}, \mathbf{2}, \mathbf{3}, \mathbf{4}, \mathbf{5}, \mathbf{6}\}$

3 / Life duration: $\boldsymbol{\Omega} = \mathbb{R}_+$

Definition 02

The set of outcomes of an experiment that verify a given property is called an event. It is therefore a part A of $\boldsymbol{\Omega}\big(\boldsymbol{A} \in \boldsymbol{P}(\boldsymbol{\Omega})\big)$.

Example:

1° / Getting tails by tossing a piece of money.

2° / Obtain a sum ≥ 7 by simultaneously throwing two pieces of money.

* The set Ω is an event called a certain event.

* The set ∅ is an event called an impossible event.

* Two events A and B are said to be incompatible if $A \cap B = \emptyset$

Definition 03

An elementary (simple) event is a part of Ω reduced to a single element (E={w}).

Definition 04

We call a complete system of events any $P-ulpet(A_1, A_2, A_3 \dots, A_p)$ of events or sequence of events $(A_n)_{n\geq 1}$ two by two incompatible distinct from ∅ such that:

$$\bigcup_{k=1}^{p} A_k = \boldsymbol{\Omega}$$

Axiome 01 :

$$0 \leq P(w_i) \leq 1$$

Axiome 02 :

$$\sum_{i=1}^{\infty} P(w_i) = 1$$

2.1.1. Probability:

Let Ω be a probability set, A is a subset of Ω

$$\mathbb{P}: \left(\boldsymbol{P}(\boldsymbol{\Omega})\right) - - - - - \rightarrow [\mathbf{0}, \mathbf{1}]$$

$$\mathbf{A} - - - - \rightarrow \mathbb{P}(\boldsymbol{A}) = \frac{\boldsymbol{card}(\boldsymbol{A})}{\boldsymbol{card}(\boldsymbol{\Omega})}$$

We can write

$$\mathbb{P}(A) = \frac{Number\ of\ favourable\ cases}{Number\ of\ possible\ cases}$$

2.1.2. Equiprobability :

If all the elementary events have the same probability, we say that there is equiprobability.

Example:

In a throw of the dice

$$\mathbb{P}_{\mathbf{1}} = \mathbb{P}_{\mathbf{2}} = \mathbb{P}_{\mathbf{3}} = \mathbb{P}_{\mathbf{4}} = \mathbb{P}_{\mathbf{5}} = \mathbb{P}_{\mathbf{6}} = \frac{\mathbf{1}}{\mathbf{6}}$$

2.1.3. Tribute:
Let be a fundamental set and is another set, B a subset. We say that is a tribe (also called an algebra) if it satisfies the axioms :

1° / $\Omega \subset \mathcal{A}$

2° / $B \in \mathcal{A} \Rightarrow B^c \in \mathcal{A}$

3° / Si (B_k). is a sequence of elements belonging to $\mathcal{A}$ then

$$\bigcup_{k=1}^{p} B_k \epsilon \mathcal{A}$$

Example

$\Omega = \{\{a\},\{b\},\{c\}\}$

$\mathcal{A}_1 = \{\emptyset,\{a\},\{b\},\{c\}\}$ **is not a tribute**

$\mathcal{A}_2 = \{\emptyset,\{a\},\{b,c\},\{a,c\},\{b,a\},\{c\},\{b\},\{a,b,c\}\}$ **is a tribute**

2.2 Probabilised space

2.2.1. Kolmogrov axiomatics

Let Ω be a set. A tribe on Ω We call probability on (Ω,A) any application

Checking
1° / $\mathbb{P}(\Omega) = \mathbf{1}$

2° $(A_i)_{i\epsilon I}$ is a family of two distinct elements of A (i.e.)

$A_i \cap A_j = \emptyset . \forall\, i, j$

$$\mathbb{P}\left(\bigcup_{k=1}^{p} A_i\right) = \sum_{i \in I} \boldsymbol{P}(A_i)$$

Definition: The triplet (Ω,A,P) is called a probability space. $(\Omega, \mathcal{A}, \mathbb{P})$

Definition: The cardinal of a set Ω, denoted card (Ω), represents its number of elements, for example Ω={1,2,5,8}→ card (Ω)=4

2.2.2. Elementary probability

1° / $\mathbb{P}(\emptyset) = 0$

2° $\mathbb{P}(\bar{A}) = 1 - \mathbb{P}(A)$

3° / $\mathbb{P}(A) \leq \mathbb{P}(B)\ si\ A \subset B$

4° $\mathbb{P}(A \cup B) = \mathbb{P}(A) + \mathbb{P}(B) - \mathbb{P}(A \cap B)$

Definition (Probability (finite case)) A probability P is a function of P(Ω) in [0; 1] such that :

- P(Ω) = 1;
- P(A U B) = P(A) + P(B) si A et B sont disjoints
- $\bar{\bar{A}} = A$, $\bar{\emptyset} = \Omega$, $\bar{\Omega} = \emptyset$

- $A \cap \overline{A} = \emptyset$, $A \cup \bar{A} = \Omega$
- $\overline{(A \cap B)} = \overline{A} \cup \overline{B}$
- $\overline{(A \cup B)} = \overline{A} \cap \overline{B}$
- si $(A \subset B)$ alors $\overline{A} \subset \overline{B}$
- $\text{card}(\bar{A}) = \text{card}(\Omega) - \text{card}(A)$

Probability of events.
The probability of an event is a numerical value that represents the proportion of times the event will **occur** when the experiment is repeated under **identical conditions**. We can deduce from this definition that a probability must be between 0 and 1 and that the probability of an event is the sum of the probabilities of each of the elementary events that make it up. Finally, the sum of the probabilities of all the elements of Ω is 1.

Space of observables :
In each situation, the space of observables (or outcomes) is therefore denoted Ω, while the elements of Ω, called observations, will be denoted ω. Let's look at some examples of how we model a random experiment.

Definition:
Any set of finite cardinal is called a finite probability space (it is also called a state space, or universe Ω).
An event is a set of outcomes (a subset of the universe) of a random experiment. Since an event is a statement about the outcome of an experiment, we need to be able to say, for any outcome in the universe, whether the event happens or not.

Example 01:
Consider a 6-sided die, numbered 1 to 6. The set of possible outcomes of a throw is called Ω = {1; 2; 3; 4; 5; 6}. Assuming that the die is balanced, we associate the probability
probability 1/6 to each result. We therefore think of a probability as a function P on Ω such that :
P(1) + P(2) + P(3) + P(4) + P(5) + P(6) = 1:

Example 02:
We consider the same die, except that on the sixth face the number 6 is replaced by 5. There are therefore two sides with 5 on them. The set of results is different: A = {1; 2; 3; 4; 5}.
As the die is balanced, faces 1, 2, 3 and 4 each have a probability of 1/6 of coming up,
P(5) = 2/6.

For example, si Ω = {1; 2; 3},

Then P(Ω) = {Ø; {1} ; {2} ; {3} ; {1; 2} ; {1; 3} ; {2; 3} ; {1; 2; 3}}

2.4. Independent conditional probability:
Let (Ω,A,P) be a probabilised space, the intersection of two events denoted A∩B where only if both events A and B are realised. However we can be interested in the realisation of event A knowing that event B has already been realised, if this event has a non-zero probability i.e. we are interested in the conditional probability knowing B.
The conditional probability knowing B is the application of ∈[0,1] defined by :

$$\forall \boldsymbol{A} \in \boldsymbol{C}, \mathbb{P}(\boldsymbol{A} \mid \boldsymbol{B}) = \boldsymbol{P}_{\boldsymbol{B}}(\boldsymbol{A}) = \frac{\boldsymbol{P}(A \cap B)}{\boldsymbol{P}(\boldsymbol{B})}$$

Incompatible event:
A and B are two incompatible events if and only if $(A \cap B)=\emptyset$
Example:
A candidate takes a competition
A: successful
B: failure
C: absence
A and C are incompatible.

- Incompatible events A and B:

Event A will not occur if event B occurs:

$$\mathbb{P}(\boldsymbol{A} \mid \boldsymbol{B}) = \mathbf{0}$$

$$\mathbb{P}(\boldsymbol{A} \cup \boldsymbol{B}) = \boldsymbol{P}(\boldsymbol{A}) + \boldsymbol{P}(\boldsymbol{B})$$

Example 1:
Two dice are thrown and the two events are considered:
- A: obtain an odd number on both dice ;
- B: the sum of the points obtained on the two dice is an odd number..

$$A = \{1,3\},\{1,5\},\{5,3\},\{1,1\},\{3,3\},\{5,5\}$$

$$B = \{1,2\},\{1,4\},\{1,6\},\{3,2\},\{5,2\},\{3,4\},\{3,6\}\{4,5\},\{6,5\}$$

$$(\mathrm{A} \cap \mathrm{B}) = \emptyset \Rightarrow \mathrm{A\ et\ B\ sont\ disjoint}$$

2.5. Independence of events :
2.5.1. Definition
Event A is independent of event B if the probability of occurrence of event A is not modified by information concerning the occurrence of event B.Event A is independent of event B if the probability of occurrence of event A is not modified by information concerning the occurrence of event B.

Meaning

$$P(\mathrm{A} \cap \mathrm{B}) = P(A) \times P(B) \Leftrightarrow A \perp B$$

Then we say that A is independent of B

The principle of compound probabilities leads to :

$\mathbb{P}(A \mid B) = P(A)$

Compound probability theory
This principle translates into Poincaré's formula
$P(A_1 \cap A_2 \cap \cdots \cap A_n) = P(A_1) \cdot P(A_2 \mid A_1) \cdot P(A_3 \mid A_1 \cap A_2) \cdots P(A_n \mid A_1 \cap \ A_2 \cdots \cap A_{n-1})$

Example 01: (examination)
There are 6 machines in a box, 4 of them are in working order and 2 are broken. Three machines are chosen at random, one after the other.
Find the probability that the 1st and 3rd are working and the 2nd is a defect.

Solution:
Bi: "the i^{th} machine selected is in good condition".
We are looking for the probability $P(B_1 \cap \overline{B_2} \cap B_3)$

$$P(B_1 \cap \overline{B_2} \cap B_3) = P(B_1)P(\overline{B_2}/B_1)P(B_3/B_1 \cap \overline{B_2})$$

$$=\frac{4}{6}\times\frac{2}{5}\times\frac{3}{4}=\frac{1}{5}=0,2$$

Example 2:
We have 7 balls, 4 white and 3 black.
Select 3 balls in order and without a restart.
Calculate the probability that the three balls are white.
Solution :
Consider
B1: a white ball on the 1st selection.
B2: a white ball on the 2nd selection.
B3: a white ball on the 3rd selection.

$$\boldsymbol{P(B_1\cap B_2\cap B_3)=P(B_1)\times P(B_2/B_1)\times P(B_3/(B_1\cap B_2))}$$
$$=\mathbf{4}/_{\mathbf{7}}\times\mathbf{3}/_{\mathbf{6}}\times\mathbf{2}/_{\mathbf{5}}=\mathbf{4}/_{\mathbf{35}}$$

2.5.2 Pairwise independence

To judge B_1, B_2, B_3 to be independent of each other, it is necessary that

$$\boldsymbol{P(B_1\cap B_2)=P(B_2\cap B_3)=P(B_1\cap B_3)}$$

2.5.3 Mutual independence

The events $A_i, i\in(1,2,\dots,n)$ are mutually independent if for any part I of the set of indices we have :

$$\boldsymbol{P\left(\bigcap_{i\in I}A_i\right)=\prod_{i\in I}P(A_i)}$$

Example:
We throw two dice and consider the events:
A: The first die has an even face ;
B: The second die has an odd side;
C: The sum of the dots on the two faces is even

$\boldsymbol{A=\{2,4,6\}}$

$\boldsymbol{B=\{1,3,5\}}$

.

+	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

$P(A) = \frac{3}{6} = \frac{1}{2}, P(B) = \frac{1}{2}, P(C) = \frac{18}{36}$

$P(A \cap B)$ =purple

$P(A \cap C)$ =blue

$P(C \cap B)$ =green

$$P(A \cap B) = P(A \cap C) = P(C \cap B) = \frac{9}{36} = \frac{1}{4} \Rightarrow A, B, C \ are\ independent\ two\ by\ two$$

$P(A \cap B \cap C) = 0 \neq P(A) \times P(B) \times P(C)$

Events A, B and C are not mutually independent:

2.5.5. Bayes theory

Definition

The subsets $A_1, A_2 \cdots A_n$ of the set Ω are said to form a partition of Ω if
1°/ $\bigcup_{i=1}^{n} A_i = \boldsymbol{\Omega}$

2°/ $A_i \cap A_j = \emptyset, \forall\, 1 \leq i, j \leq n$

Theorem: Let (Ω,A,P) be a probabilized space A_i a partition of Ω then we have the following realization $\forall\, \boldsymbol{B} \in \boldsymbol{\mathcal{A}}$

1°/ Total probability formula :

$$\boldsymbol{P}(\boldsymbol{B}) = \sum_{\boldsymbol{i}=\boldsymbol{1}}^{\boldsymbol{n}} (\boldsymbol{B} \cap A_i) = \sum_{\boldsymbol{i}=\boldsymbol{1}}^{\boldsymbol{n}} \boldsymbol{P}(\boldsymbol{B}/A_i) \times \boldsymbol{P}(A_i)$$

2°/ Bayes formula :

$$\boldsymbol{P}(A_i/\boldsymbol{B}) = \frac{\boldsymbol{P}(\boldsymbol{B}/A_i) \times \boldsymbol{P}(A_i)}{\boldsymbol{P}(\boldsymbol{B})} = \frac{\boldsymbol{P}(\boldsymbol{B}/A_i) \times \boldsymbol{P}(A_i)}{\sum_{\boldsymbol{i}=\boldsymbol{1}}^{\boldsymbol{n}} \boldsymbol{P}(\boldsymbol{B}/\boldsymbol{A_i})\boldsymbol{P}(\boldsymbol{A_i})}$$

Exercise :

A series of urns contains the following balls:

$$U_1 = \begin{vmatrix} 2B \\ 1N \end{vmatrix}, U_2 = \begin{vmatrix} 3B \\ 4N \end{vmatrix}, U_3 = \begin{vmatrix} 5B \\ 2N \end{vmatrix}$$

A white ball has been drawn from U, what is the probability of it being drawn from $\boldsymbol{U_1}$

$$P(U_1/B) = \frac{P(U_1 \cap B)}{P(B)} = \frac{P(B/U_1) \times P(U_1)}{P(B)}$$

$$\frac{P(B/U_1) \times P(U_1)}{P(B \cap U_1) + P(B \cap U_2) + P(B \cap U_3)}$$

$$\frac{P(B/U_1) \times P(U_1)}{P(B/U_1) \times P(U_1) + P(B/U_2) \times P(U_2) + P(B/U_3) \times P(U_3)}$$

$$\frac{{}^2/_3 \times {}^1/_3}{({}^2/_3 \times {}^1/_3) + ({}^3/_7 \times {}^1/_3) + \left({}^5/_7 \times {}^1/_3\right)}$$